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$D = 4$ pure spinor superstring and $N = 2$ strings

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Abstract: We study the compactification of the pure spinor superstring down to four dimensions. We find that the compactified string is described by a conformal invariant system for both the four dimensional and for the compact six dimensional variables. The four dimensional sector is found to be invariant under a non-critical $N = 2$ superconformal transformations.

Keywords: Superstrings and Heterotic Strings, Conformal Field Models in String Theory.

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1. Introduction

The pure spinor formalism of the superstring has solved the longstanding problem of the covariant quantization of the superstring [1]. This formalism has passed many tests since it was formulated. It describes correctly the superstring spectrum in the light cone gauge [2], in semi light cone gauge [3] (see also [4]) and in a manifestly ten dimensional covariant manner [5]. It has also been possible to compute amplitudes by using this formalism. Tree-level amplitudes were shown to coincide with the RNS result in [6] and loop amplitudes were defined and used to prove certain non renormalization theorems for low energy effective action terms in [7].

The formalism has been used to construct quantizable sigma model actions in curved backgrounds. The action for a generic supergravity/super Yang-Mills background was studied in [8] at the classical level and its conformal invariance at the quantum level was verified in [9]. Backgrounds supporting Ramond Ramond fluxes can also be constructed in this framework. Namely, the $\mathrm{AdS}_5 \times \mathrm{S}^5$ case was classically studied in [10] and quantum mechanically in [11].

Attempts to extract a manifest space time supersymmetry from the RNS formalism is only possible if the string is compactified to four [12] or to six dimensions [13]. In ten dimensions is possible to preserve a U(5) subgroup of SO(10) [14]. Since the pure spinor formalism is manifestly supersymmetric in ten dimensions, it is tempting to find a suitable compactification and relate the resulting theory to a hybrid string. We pursue this objective for the four dimensional version of the hybrid superstring. We will find that the four dimensional pure spinor string has the same matter world-sheet variables as the hybrid superstring. They differ in the ghosts variables. In the pure spinor case, the ghosts are given by an unconstrained c-number Weyl spinor, while for the hybrid string the ghost is a chiral boson. As for the hybrid string, our compactified pure spinor string will be invariant under $N = 2$ superconformal transformations but the corresponding algebra has a different central charge. During the conclusion of this work, we have noticed the appearance of the papers [15, 16] and [17] with some overlapping with our results.

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In the next section 2 we will compactify the ten dimensional pure spinor string to preserve four dimensional supersymmetry. In section 3 we will find an $N = 2$ superconformal structure, although the central charge turns out to be $c = -6$ for the four dimensional sector.

2. The four dimensional pure spinor superstring

In this section we will compactify the ten dimensional pure spinor superstring. We will first review the ten dimensional pure spinor string and then we will compactify down to four dimensions.

2.1 Review of the ten dimensional pure spinor string

The pure spinor string in ten dimensions is described by the free-field conformal invariant action

$$S = \int d^2z \frac{1}{2} \partial X^{\widehat{m}} \overline{\partial} X_{\widehat{m}} + p_{\widehat{\alpha}} \overline{\partial} \theta^{\widehat{\alpha}} + S_{\text{pure}} \,, \tag{2.1}$$

where $(X^{\widehat{m}}, \theta^{\widehat{\alpha}})$ with $\widehat{m} = 0, \ldots, 9$ and $\widehat{\alpha} = 1, \ldots, 16$ define the ten-dimensional superspace, $p_{\widehat{\alpha}}$ is the conjugate momentum of $\theta^{\widehat{\alpha}}$ and S_{pure} is the action for the pure spinor ghosts variables. They are a c-number ten dimensional spinor $\lambda^{\widehat{\alpha}}$ constrained by the condition $(\lambda \gamma^{\widehat{m}} \lambda) = 0$ and its conjugate $\omega_{\widehat{\alpha}}$ which is defined up to the gauge transformation $\delta \omega_{\widehat{\alpha}} = \Lambda_{\widehat{m}} (\gamma^{\widehat{m}} \lambda)_{\widehat{\alpha}}$, here the $\gamma^{\widehat{m}}$ are the symmetric 16×16 ten-dimensional Pauli matrices. One can check that the pure spinor constraint implies that only 11 out of 16 components of $\lambda^{\widehat{\alpha}}$ are independent. Similarly the gauge transformation for $\omega_{\widehat{\alpha}}$ implies that 11 out of its sixteen components are independent. Therefore, the action of (2.1) is conformal invariant. That is, the central charge vanishes since we have 32 bosons and 32 fermions in the world-sheet action of (2.1).

Physical states are defined as ghost number one vertex operators in the cohomology of the BRST operator $Q = \oint \lambda^{\widehat{\alpha}} d_{\widehat{\alpha}}$, where

$$d_{\widehat{\alpha}} = p_{\widehat{\alpha}} - \frac{1}{2} \left(\gamma^{\widehat{m}} \theta \right)_{\widehat{\alpha}} \partial X_{\widehat{m}} - \frac{1}{8} \left(\gamma^{\widehat{m}} \theta \right)_{\widehat{\alpha}} (\theta \gamma_{\widehat{m}} \partial \theta) \,,$$

is the world-sheet generator of superspace translations. Since the $d_{\widehat{\alpha}}$'s satisfy the OPE

$$d_{\widehat{\alpha}}(y) d_{\widehat{\beta}}(z) \to -\frac{1}{(y-z)} \gamma^{\widehat{m}}_{\widehat{\alpha}\widehat{\beta}} \Pi_{\widehat{m}}(z) \,,$$

where $\Pi^{\widehat{m}} = \partial X^{\widehat{m}} + \frac{1}{2} (\theta \gamma^{\widehat{m}} \partial \theta)$, it is trivial to check the nilpotence of the BRST charge. A physical state is given by a vertex operator V which is annihilated by Q up to an operator of the form $Q\Omega$. At ghost number one, massless states of the open strings are shown to describe super Maxwell multiplet in ten dimensions after using BRST conditions. Namely, a ghost number one vertex operator is of the form $V = \lambda^{\widehat{\alpha}} A_{\widehat{\alpha}}(X, \theta)$, where $A_{\widehat{\alpha}}(X, \theta)$ is superfield which depends on the zero modes of (X, θ) only. It guarantees that V is describing massless fields. It is easy to obtain the equation of motion derived by the condition $QV = 0$. It is $\lambda^{\widehat{\alpha}} \lambda^{\widehat{\beta}} D_{\widehat{\alpha}} A_{\widehat{\beta}} = 0$, where $D_{\widehat{\alpha}}$ is the space time supersymmetric

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covariant derivative. This equation is solved by $D_{(\widehat{\alpha}}A_{\widehat{\beta})} = \gamma^{\widehat{m}}_{\widehat{\alpha}\widehat{\beta}}A_{\widehat{m}}$. The gauge invariance is obtained from the BRST invariance $\delta V = Q\Omega$ which implies $\delta A_{\widehat{\alpha}} = D_{\widehat{\alpha}}\Omega$. After using this gauge invariance one can show that the θ independent part of the superfield $A_{\widehat{m}}$ is the photon and the term linear in θ involves the photino.

So far, we have no discussed the pure spinor constraint. We have only used its main property to check the nilpotence of the BRST charge. In order to verify Lorentz invariance, to construct massive states and to properly define loop amplitudes, we need to solve the pure spinor constraint. The pure spinor ghosts can only appear in combinations that preserve the gauge invariance $\delta\omega_{\widehat{\alpha}} = \Lambda_{\widehat{m}}(\gamma^{\widehat{m}}\lambda)_{\widehat{\alpha}}$. At ghost number zero, the only possible choices are $\omega_{\widehat{\alpha}}\partial\lambda^{\widehat{\alpha}}$, $\lambda^{\widehat{\alpha}}\omega_{\widehat{\alpha}}$ and $(\omega\gamma^{\widehat{m}\widehat{n}}\lambda)$. They are respectively the stress tensor T_λ, the ghost number current J and the Lorentz current $N^{\widehat{m}\widehat{n}}$ for the pure spinor ghosts. In order to compute the algebras for these generators one has to temporally break Lorentz covariance. For example one can choose to preserve a U(5) subgroup of SO(10) as in [1]. Although the computation is not covariant, the result is. The OPE algebra one obtains is

$$\begin{aligned} N^{\widehat{m}\widehat{n}}(y)N^{\widehat{p}\widehat{q}}(z) &\to \frac{3}{(y-z)^2}\eta^{\widehat{m}[\widehat{p}}\eta^{\widehat{q}]\widehat{n}} + \frac{1}{(y-z)}\left(\eta^{\widehat{m}[\widehat{p}}N^{\widehat{q}]\widehat{n}} - \eta^{\widehat{n}[\widehat{p}}N^{\widehat{q}]\widehat{m}}\right), \\ J(y)J(z) &\to -\frac{4}{(y-z)^2}, \qquad T_\lambda(y)T_\lambda(z) \to \frac{22}{2(y-z)^4} + \frac{2T_\lambda(z)}{(y-z)^2} + \frac{\partial T_\lambda(z)}{(y-z)}, \\ T_\lambda(y)J(z) &\to \frac{8}{(y-z)^3} + \frac{J(z)}{(y-z)^2} + \frac{\partial J(z)}{(y-z)}. \end{aligned} \tag{2.2}$$

This algebra determine the current $N^{\widehat{m}\widehat{n}}$ to be the Lorentz generator for the pure spinor variables and give rise to the right double pole coefficient in the $N^{\widehat{m}\widehat{n}}(y)N^{\widehat{p}\widehat{q}}(z)$ OPE. The central charge is 22 which cancel the -22 contribution from the matter variables and leads to an anomaly of $+8$ in the ghost number current.

2.2 Compactification to four dimensions

Now we go to four dimensions. First of all, we need to write the SO(10) representations such that we can extract SO(4) representations. By using the SU(4) notation of [18], the pure spinor $\lambda^{\widehat{\alpha}}$ can be decomposed as

$$\lambda^{\widehat{\alpha}} = \left(\lambda^\alpha_A, \overline{\lambda}^{\dot{\alpha}A}\right), \tag{2.3}$$

where $\alpha, \dot{\alpha} = 1, 2$, $A = 1, \ldots, 4$. The pure spinor condition $(\lambda\gamma^{\widehat{m}}\lambda) = 0$ decomposes in the following conditions

$$\lambda^\alpha_A\overline{\lambda}^{\dot{\alpha}A} = 0, \qquad \epsilon_{\alpha\beta}\lambda^\alpha_A\lambda^\beta_B + \frac{1}{2}\epsilon_{\dot{\alpha}\dot{\beta}}\epsilon_{ABCD}\overline{\lambda}^{\dot{\alpha}C}\overline{\lambda}^{\dot{\beta}D} = 0, \tag{2.4}$$

the first comes from $\widehat{m} = 0, \ldots, 3$ and the second from the remaining directions.

To extract a four dimensional spinor, we make one more decomposition. We use $SU(3) \times U(1)$ instead of SU(4) to write

$$\lambda^\alpha_A = (\lambda^\alpha, \lambda^\alpha_I), \qquad \overline{\lambda}^{\dot{\alpha}A} = \left(\overline{\lambda}^{\dot{\alpha}}, \overline{\lambda}^{\dot{\alpha}I}\right), \tag{2.5}$$

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where $I = 1, \dots, 3$. The non trivial pure spinor conditions in (2.4) become

$$\begin{aligned} \lambda^{\alpha}\overline{\lambda}^{\dot{\alpha}} + \lambda^{\alpha}_{I}\overline{\lambda}^{\dot{\alpha}I} &= 0\,, \\ \epsilon_{\alpha\beta}\lambda^{\alpha}\lambda^{\beta}_{I} + \frac{1}{2}\epsilon_{\dot{\alpha}\dot{\beta}}\epsilon_{IJK}\overline{\lambda}^{\dot{\alpha}J}\overline{\lambda}^{\dot{\beta}K} &= 0\,, \\ \epsilon_{\alpha\beta}\lambda^{\alpha}_{I}\lambda^{\beta}_{J} + \epsilon_{\dot{\alpha}\dot{\beta}}\epsilon_{IJK}\overline{\lambda}^{\dot{\alpha}}\overline{\lambda}^{\dot{\beta}K} &= 0\,. \end{aligned} \tag{2.6}$$

Since pure spinors in four dimensions have only two components [19], we expect that a non trivial solution to these conditions is

$$\lambda^{\alpha}_{A} = (\lambda^{\alpha}, \lambda^{\alpha}_{I})\,, \qquad \overline{\lambda}^{\dot{\alpha}A} = \left(0, \overline{\lambda}^{\dot{\alpha}I}\right). \tag{2.7}$$

The field λ^{α} is unconstrained and the remaining components are constrained according to (2.6). After putting $\overline{\lambda}^{\dot{\alpha}} = 0$ in the third condition in (2.6) one obtains

$$\epsilon_{\alpha\beta}\lambda^{\alpha}_{I}\lambda^{\beta}_{J} = 0\,, \tag{2.8}$$

which implies λ^{2}_{I} is proportional to λ^{1}_{I} (it will be called as λ_I). Then we have 11 components for a pure spinor, namely $(\lambda^{\alpha}, \lambda_I, \overline{\lambda}^{\dot{\alpha}I})$.

Now we examine if we can gauge fix some components of the conjugate pure spinor variable $\omega_{\widehat{\alpha}}$. Recall that it is defined up to the gauge transformation $\delta\omega_{\widehat{\alpha}} = \Lambda_{\widehat{m}}(\gamma^{\widehat{m}}\lambda)_{\widehat{\alpha}}$. We note first that only five of the ten gauge parameters $\Lambda_{\widehat{m}}$ are independent. We write $\Lambda_{\widehat{m}} = (\Lambda_m, \Lambda_I, \Lambda_{IJ})$, where Λ_{IJ} is antisymmetric. Only two of the four Λ_m are independent and only three of the six $(\Lambda_I, \Lambda_{IJ})$, here we can choose $\Lambda_{IJ} = 0$. Using this gauge invariance we can gauge fix to zero the components $\overline{\omega}_{\dot{\alpha}}$ and ω^{I}_{2}. In fact,

$$\delta\overline{\omega}_{\dot{\alpha}} = \Lambda_m \sigma^{m}_{\beta\dot{\alpha}}\lambda^{\beta}\,, \qquad \delta\omega^{I}_{\alpha} = \Lambda_m\sigma^{m}_{\alpha\dot{\beta}}\overline{\lambda}^{\dot{\beta}I} - \epsilon_{\alpha\beta}\Lambda^{I}\lambda^{\beta}\,,$$

by using the two independent Λ_m to gauge fix $\overline{\omega}_{\dot{\alpha}}$ and the three Λ^I to gauge fix the three ω^{I}_{2}.

The BRST current can be written as

$$j_{BRST} = \lambda^{\widehat{\alpha}}d_{\widehat{\alpha}} = \lambda^{\alpha}d_{\alpha} + \lambda^{\alpha}_{I}d^{I}_{\alpha} + \overline{\lambda}^{\dot{\alpha}I}\overline{d}_{\dot{\alpha}I}\,,$$

where the ten-dimensional world-sheet generator of superspace translations $d_{\widehat{\alpha}}$ is decomposed as $(d_{\alpha}, \overline{d}_{\dot{\alpha}}, d^{I}_{\alpha}, \overline{d}_{\dot{\alpha}I})$ and λ^{α}_{I} constrained as (2.8). Physical states are in the cohomology of $Q = \oint j_{BRST}$, for those states which are independent of the compactification variables the BRST current is simply equal to $\lambda^{\alpha}d_{\alpha}$. As was noted in [15], d_{α} can be mapped to p_{α}. Therefore, the physical states depend on $(X^m, \overline{\theta}, \overline{p})$ only and describe the chiral sector of the open superstring field theory of [15].

3. $N = 2$ superconformal invariance

In this section we will study the $N = 2$ superconformal invariance of the four dimensional part of the pure spinor superstring action of (2.1) when it is compactified to four dimensions. In the previous section we solved the pure spinor constraint in a form which is suitable for the compactification we want to perform. It remains what to do with the matter part of the action. It is natural to split $X^{\widehat{m}} = (X^m, Y^I, \overline{Y}^I)$ with $m = 0, \ldots, 3$ and $I = 1, \ldots 3$. For the superspace variables we use the SU(3) $\times$ U(1) notation as in the previous section. The action becomes

$$S = \int d^2z \frac{1}{2}\partial X^m \overline{\partial} X_m + p_\alpha \overline{\partial}\theta^\alpha + \overline{p}_{\dot{\alpha}} \overline{\partial}\overline{\theta}^{\dot{\alpha}} + \omega_\alpha \overline{\partial}\lambda^\alpha + S_C \,, \tag{3.1}$$

where S_C stands for the compact part of the action and it is given by

$$S_C = \int d^2z \; \partial \overline{Y}^I \overline{\partial} Y_I + p^I_\alpha \overline{\partial}\theta^\alpha_I + \overline{p}^I_{\dot{\alpha}} \overline{\partial}\overline{\theta}^{\dot{\alpha}}_I + \overline{\omega}_{\dot{\alpha} I} \overline{\partial}\overline{\lambda}^{\dot{\alpha} I} + \omega^I_\alpha \overline{\partial}\lambda^\alpha_I \,, \tag{3.2}$$

where λ^α_I is constrained according to (2.8). We note that our action has vanishing central charge. Now it will be proven that both the four dimensional and the compact six dimensional part of the action have $N = 2$ superconformal invariance.

3.1 $N = 2$ $D = 4$

For the four dimensional part of the action we have eight bosons and eight fermions, then the stress tensor becomes

$$T = -\frac{1}{2}\partial X^m \partial X_m - p_\alpha \partial\theta^\alpha - \overline{p}_{\dot{\alpha}} \partial\overline{\theta}^{\dot{\alpha}} - \omega_\alpha \partial\lambda^\alpha \,,$$

and satisfies the OPE algebra

$$T(y)T(z) \rightarrow \frac{2T(z)}{(y-z)^2} + \frac{\partial T(z)}{(y-z)} \,.$$

In order to relate the pure spinor string to the hybrid formalism, one needs to show a hidden $N = 2$ superconformal invariance. One is tempted to write the twisted $N = 2$ generators to be

$$\begin{aligned} T &= -\frac{1}{2}\partial X^m \partial X_m - p_\alpha \partial\theta^\alpha - \overline{p}_{\dot{\alpha}} \partial\overline{\theta}^{\dot{\alpha}} - \omega_\alpha \partial\lambda^\alpha \,, \\ G^+ &= \lambda^\alpha d_\alpha \,, \\ G^- &= \omega_\alpha \partial\theta^\alpha + \Pi^m \left(y \sigma_m \overline{d}\right) , \\ J &= \lambda^\alpha \omega_\alpha \,, \end{aligned} \tag{3.3}$$

where Π^m is the four dimensional projection of $\Pi^{\widehat{m}}$ and $d_\alpha, \overline{d}_{\dot{\alpha}}$ come from the ten dimensional field $d_{\widehat{\alpha}}$. The field y_α comes from the ten dimensional projector field $y_{\widehat{\alpha}}$ defined as $y_{\widehat{\alpha}} = v_{\widehat{\alpha}}/(v_{\widehat{\beta}}\lambda^{\widehat{\beta}})$ with the property $y_{\widehat{\alpha}}\lambda^{\widehat{\alpha}} = 1$ [20]. Since $y_{\widehat{\alpha}}$ turns out to be a pure spinor, one can use the decomposition of the previous section to consider only y_α defined as $v_\alpha/(v_\beta \lambda^\beta)$ with v_α an arbitrary but constant c-number spinor in four dimensions and constrained by $\lambda^\alpha y_\alpha = 1$. In order to verify that the generators (3.3) satisfy the right $N = 2$ algebra one has to check that the OPE's $G^+(z)G^+(w)$ and $G^-(z)G^-(w)$ are not singular. It is easy to verify this for G^+. For G^- one has to be careful. There is a potential singular term coming from the contraction between Π^m with itself which has the form

$$\frac{1}{(z-w)} \epsilon^{\alpha\beta} y_\alpha \partial y_\beta \overline{d}_{\dot{\alpha}} \overline{d}^{\dot{\alpha}} \,.$$

Recall the definition of the projector field $y_\alpha = v_\alpha/(v_\beta \lambda^\beta)$ with v_α being a constant c-number spinor [20]. From this definition one obtains

$$\partial y_\alpha = -y_\alpha y_\beta \partial \lambda^\beta \,,$$

plugging this into the above equation one see that the potential singular term is zero since $\epsilon^{\alpha\beta} y_\alpha y_\beta = 0$.

This construction failed to be a critical system since the $J(y)J(z)$ OPE leads to a central charge of $\hat{c} = -2$ instead of the critical value of $+2$. A solution for this problem is the non minimal construction of [15].

3.2 $N = 2$ $D = 6$

For the compact six dimensional part of the action, we must to take into account the constraint (2.8) for the pure spinor component λ^α_I. The $N = 2$ generators are

$$\begin{aligned}
T_C &= -\partial Y^I \partial \overline{Y}_I - p^I_\alpha \partial \theta^\alpha_I - \overline{p}_{\dot{\alpha} I} \partial \overline{\theta}^{\dot{\alpha} I} + \overline{\omega}_{\dot{\alpha} I} \partial \overline{\lambda}^{\dot{\alpha} I} + \omega^I_\alpha \partial \lambda^\alpha_I \,, \\
G^+_C &= \overline{\lambda}^{\dot{\alpha} I} d_{\alpha I} + \lambda^\alpha_I d^I_\alpha \,, \\
G^-_C &= \overline{\omega}_{\dot{\alpha} I} \partial \overline{\theta}^{\dot{\alpha} I} + \omega^I_\alpha \partial \theta^\alpha_I + \epsilon_{IJK} \epsilon^{\alpha\beta} \Pi^I y^J_\alpha d^K_\beta + \epsilon^{IJK} \epsilon^{\dot{\alpha}\dot{\beta}} \overline{\Pi}_I \overline{y}_{\dot{\alpha} I} \overline{d}_{\dot{\beta} K} \,, \\
J_C &= \overline{\lambda}^{\dot{\alpha} I} \overline{\omega}_{\dot{\alpha} I} + \lambda^\alpha_I \omega^I_\alpha \,,
\end{aligned} \tag{3.4}$$

where Π^I and $\overline{\Pi}_I$ are the projections of $\Pi^{\widehat{m}}$ to six dimensions and both have conformal weight of $(1,0)$. As in the four dimensional case, we need to introduce a projector field $(y^I_\alpha, \overline{y}_{\dot{\alpha} I})$ defined by

$$y^I_\alpha = \frac{v^I_\alpha}{\left(\lambda^\beta_J v^J_\beta\right)} \,, \qquad \overline{y}_{\dot{\alpha} I} = \frac{\overline{v}_{\dot{\alpha} I}}{\left(\overline{\lambda}^{\dot{\beta} J} \overline{v}_{\dot{\beta} J}\right)} \,,$$

with the c-number spinors v^I_α and $\overline{v}_{\dot{\alpha} I}$ being arbitrary constants. The projectors are constrained to satisfy

$$\lambda^\alpha_I y^I_\alpha = \overline{\lambda}^{\dot{\alpha} I} \overline{y}_{\dot{\alpha} I} = 1 \,.$$

As in four dimensions, the hard to check part of the algebra is the OPE $G^-_C(z) G^+_C(w)$ which again vanishes in virtue of the properties of the projector fields. The $N = 2$ superalgebra has the central charge of $\hat{c} = -9$. One can obtain this result by temporarily break Lorentz invariance such that the only physical component for the pure spinor ghosts are $\overline{\lambda}^{\dot{\alpha} I}$ and one of the spinor components of λ^α_I (say $\lambda^1_I \equiv \lambda_I$) and their conjugates variables. Therefore, $J_C = \overline{\lambda}^{\dot{\alpha} I} \overline{\omega}_{\dot{\alpha} I} + \lambda_I \omega^I$, then $J_C(z) J_C(w) \to -9(z-w)^{-2}$.

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